

B. Math. III – Semestral Examination

Introduction to Differential Geometry

November 10, 2014

1. Let $\gamma(t)$ be a regular curve in \mathbb{R}^3 . Prove that it's curvature is:

$$\kappa = \frac{\|\ddot{\gamma} \times \dot{\gamma}\|}{\|\dot{\gamma}\|^3}$$

2. Define a smooth surface in \mathbb{R}^3 . Prove that the transition maps of a smooth surface are smooth functions.
3. Define an isometry between two smooth surfaces. Prove that a diffeomorphism $f : S_1 \rightarrow S_2$ between two smooth surfaces is an isometry if and only if for any surface patch σ of S_1 , the first fundamental form of σ and $f \circ \sigma$ are equal.
4. Compute the stereographic projection of the sphere minus a point onto a plane and find its first fundamental form.
5. Define a geodesic on a smooth oriented surface S in \mathbb{R}^3 , explaining every term used in the definition in full detail. Further, show that there are infinitely many geodesics on the cylinder around z -axis with radius one joining two points with different z co-ordinates.